Mock1

Whale120

February 2023

1 Introduction

rule : same as APMO/TMO

time : 2023/4/2

2 Problems

P1.Let p be a prime, and let $a_1, a_2, a_3, ...$ be a sequence of positive integers so that $a_n a_{n+2} = a_{n+1}^2 + p$ for all positive integers n. Show that a_{n+1} divides $a_n + a_{n+2}$ for all positive integers n.

P2. Determine all functions $f : Z \to Z$ with the property that

$$f(x - f(y)) = f(f(x)) - f(y) - 1$$

holds for all $x, y \in Z$.

P3. In an acute triangle ABC segments BE and CF are altitudes. Two circles passing through the point A and F and tangent to the line BC at the points P and Q so that B lies between C and Q. Prove that lines PE and QF intersect on the circumcircle of triangle AEF.

P4. Find all positive integers n such that there exists a sequence of positive integers a_1, a_2,. . ., a_n satisfying:

$$a_{k+1} = \frac{a_k^2 + 1}{a_{k-1} + 1} - 1$$

for every k with $2 \leq k \leq n - 1$.

P5.Let n and k be two integers with $n > k1$. There are $2n + 1$ students standing in a circle. Each student S has $2k$ neighbors - namely, the k students closest to S on the left, and the k students closest to S on the right.

Suppose that $n + 1$ of the students are girls, and the other n are boys. Prove that there is a girl with at least k girls among her neighbors.

3 Source

2015 Thailand Mathematical Olympiad P1
2015 A2
2008 G4
2009 N4
2021 C5